Vorticity, Impulse, and Wind Turbine Aerodynamics

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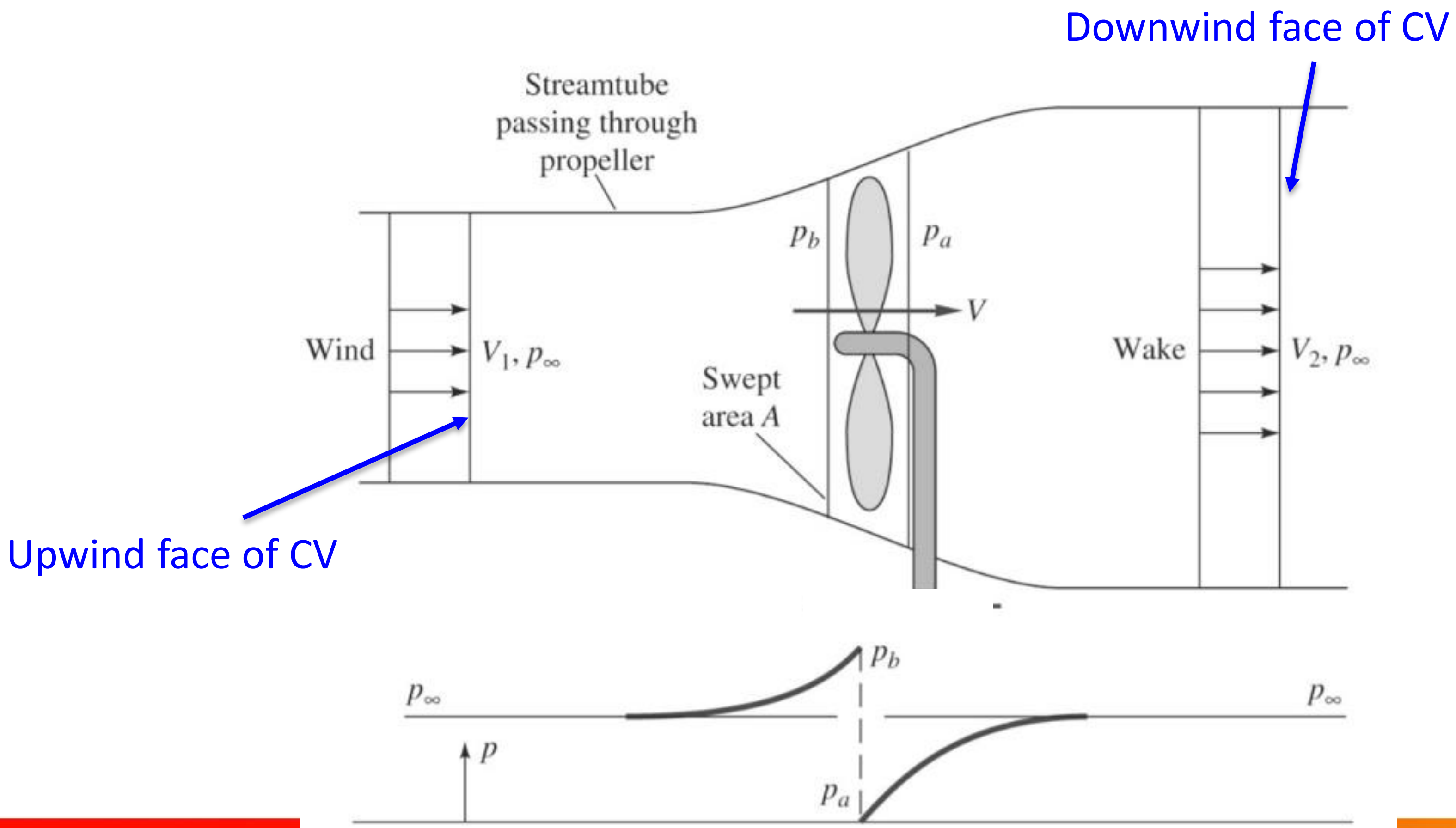
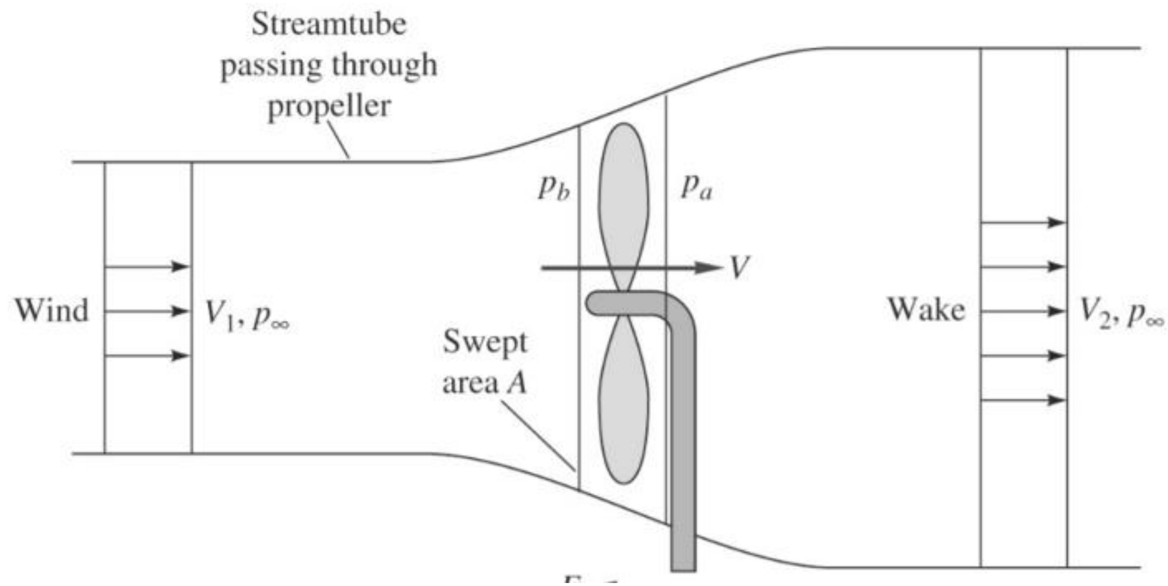
Conventional Modeling of Wind Turbines

Conventional Control Volume (CV) Analysis:

- $V_2 < V < V_1$ so kinetic energy is extracted from the wind
- $P_a > P_b$ so there is thrust, T, on the wind turbine
- Pressures upwind and downwind are equal
- Conservation of mass, momentum, angular momentum and energy are used to predict performance.



Conventional Modeling of Wind Turbines



The turbine thrust T and power P are given by

$$C_T = \frac{T}{\frac{1}{2}\rho V_1^2 A} = 4a(1-a) \qquad C_P = \frac{P}{\frac{1}{2}\rho V_1^3 A} = 4a(1-a)^2$$

where the axial induction factor a is

$$a = 1 - V/V_1$$

$C_{P,max} = 16/27$ when $a = 1/3$. This is the Betz-Joukowsky limit

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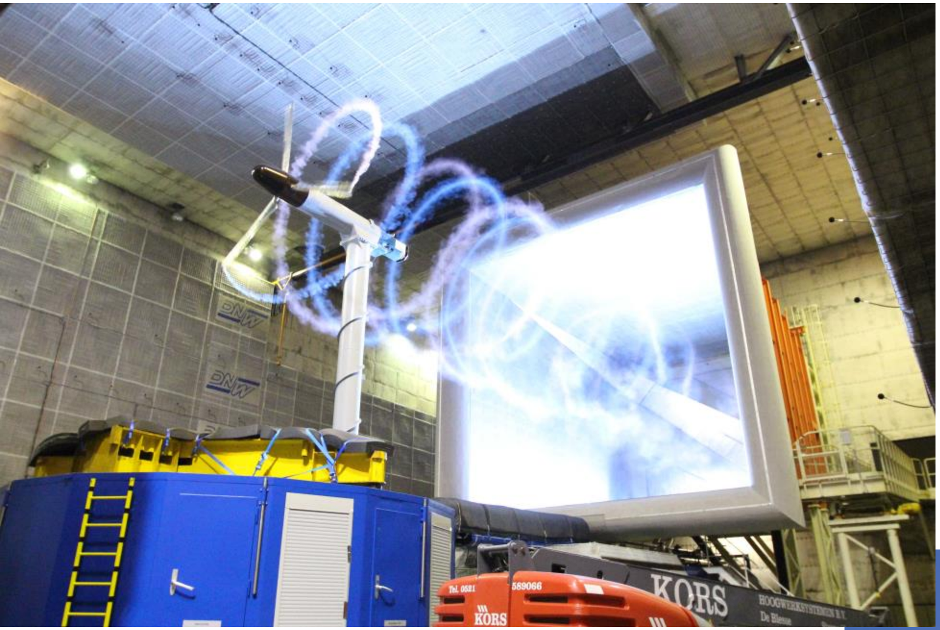
The Importance of Vorticity



Trailing vortices in a wind tunnel experiment http://www.mexnext.org/. Image provided by E.C.N., The Netherlands.

Trailing vortices behind an aircraft https://imgur.com/gallery/1vLQ1vj

Vorticity is important so why does it not appear in the basic equations for wind turbine thrust and torque

The Vortical Impulse

The impulse within a volume of fluid, V, is

$$\int_V \mathbf{x} \times \mathbf{\Omega} dV$$

where $\boldsymbol{x}$ is the position vector and $\mathbf{\Omega}$ is the vorticity. Impulse was used by Kelvin (1867) and Thomson (1892) to study vortex rings. Starting with the normal control volume (CV) form of the Reynolds transport theorem for momentum, the analysis proceeds by removing pressure. This makes impulse formulations useful for analyzing particle image velocimetry and similar velocity field data, Limacher et al. (2018, 2019).

Limacher, E., Morton, C., & Wood, D. (2018). Generalized derivation of the added-mass and circulatory forces for viscous flows. *Physical Review Fluids*, *3*(1), 014701.

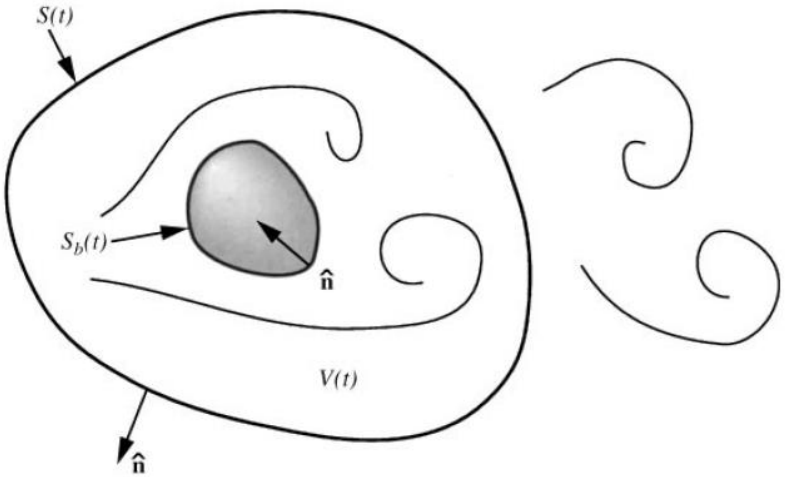
Limacher, E., Morton, C., & Wood, D. (2019). On the calculation of force from PIV data using the generalized added-mass and circulatory force decomposition. *Experiments in Fluids*, *60*(1), 4.

The Impulse Equation for Force

From Equations (2) and (3) of Noca et al. (1997) and Equations (3.55) and (3.56) of Noca (1997), the force, $\mathbf{F}$, on a three-dimensional body submerged in an incompressible fluid flow is related to the velocity and vorticity fields by

$$\frac{\mathbf{F}}{\rho} = -\frac{1}{2}\frac{d}{dt}\int_V \mathbf{x}\times\mathbf{\Omega}dV + \frac{1}{2}\int_S \mathbf{n}U^2 dS - \int_S \mathbf{n}\cdot\mathbf{U}\mathbf{U}dS +$$
$$\frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{U}(\mathbf{x}\times\mathbf{\Omega})dS - \frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{\Omega}(\mathbf{x}\times\mathbf{U})dS.$$

ρ is the air density
V is the control volume (CV)
S is the surface of the CV
$\mathbf{x}$ is the position vector
$\mathbf{n}$ is the outward facing normal from the CV
Ω is the vorticity
$\mathbf{U}$ is the velocity



Noca, F., Shiels, D., & Jeon, D. (1997). Measuring instantaneous fluid dynamic forces on bodies, using only velocity fields and their derivatives. *Journal of Fluids and Structures*, *11*(3), 345-350.
Noca, F. (1997). *On the evaluation of time-dependent fluid-dynamic forces on bluff bodies* (Doctoral dissertation, California Institute of Technology).

The Terms in the Impulse Equation for Force

$-\frac{1}{2}\frac{d}{dt}\int_V \mathbf{x}\times\mathbf{\Omega}dV$ The rate of change of impulse $= 0$ in steady flow

$\frac{1}{2}\int_S \mathbf{n}U^2 dS$ The kinetic energy (KE) at S

$-\int_S \mathbf{n}\cdot\mathbf{U}\mathbf{U}dS$ The conventional momentum deficit

$\frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{U}(\mathbf{x}\times\mathbf{\Omega})dS$ The "first vortex term".

$\frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{\Omega}(\mathbf{x}\times\mathbf{U})dS$ The second vortex term.

$$\frac{\mathbf{F}}{\rho} = -\frac{1}{2}\frac{d}{dt}\int_V \mathbf{x}\times\mathbf{\Omega}dV + \frac{1}{2}\int_S \mathbf{n}U^2 dS - \int_S \mathbf{n}\cdot\mathbf{U}\mathbf{U}dS + \frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{U}(\mathbf{x}\times\mathbf{\Omega})dS - \frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{\Omega}(\mathbf{x}\times\mathbf{U})dS.$$

Bound Vorticity and Force

Lifting bodies are considered as "bound vortices" as the lift is determined by the integral of $\rho \mathbf{U} \times \mathbf{\Omega}$ along the blade.
ρ is the air density, $\mathbf{U}$ is the velocity, and $\mathbf{\Omega}$ is the bound vorticity.
This is one version of the "Kutta-Joukowsky" theorem.
The bound vortex at the tip of a wing or a blade becomes a trailing vortex.

http://www.simpleplanes.com/a/kwJH9Y/Wind-Turbine

The Case of large Number of Blades, *N*

The turbine thrust T is given by

$$
\begin{aligned}
\frac{T}{\rho} =& \pi \int_0^1 v^2 x dx - \pi \int_0^1 a^2 x dx + 2\pi \int_0^1 (w^2 + \lambda w x) x dx + \\
& \frac{1}{2} \int_0^1 (1-a)\Omega_c x dx - \frac{1}{2} \int_0^1 \Omega_n (w + \lambda x) x dx \\
=& 2\pi \int_0^1 (w^2 + \lambda w x) x dx + \frac{1}{2} \int_0^1 (1-a)\Omega_c x dx - \\
& \frac{1}{2} \int_0^1 \Omega_n (w + \lambda x) x dx
\end{aligned}
$$

a - axial induction factor
v - radial velocity
w - circumferential velocity
x - radial co-ordinate
X - tip radius of blade
$\lambda = \omega X / V_1$ tip speed ratio
ω - angular velocity of blades

For idealized models of wind turbines, many of these terms cancel and it is possible to define the conditions under which the conventional thrust equation

$$C_T = \frac{T}{\frac{1}{2}\rho V_1^2 A} = 4a(1-a)$$

is accurate.

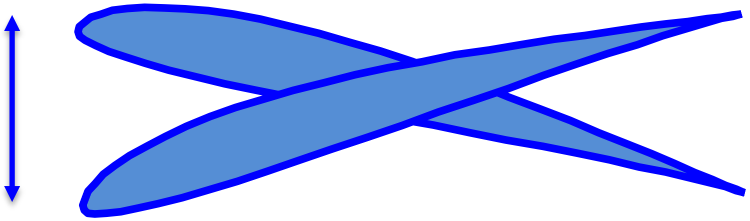
What about small *N*? What about unsteady flow?

The impulse equation (again)

$$\frac{\mathbf{F}}{\rho} = -\frac{1}{2}\frac{d}{dt}\int_V \mathbf{x}\times\boldsymbol{\Omega}dV + \frac{1}{2}\int_S \mathbf{n}U^2 dS - \int_S \mathbf{n}\cdot\mathbf{U}\mathbf{U}dS - \frac{1}{2}\int_S \mathbf{n}\cdot\mathbf{U}(\mathbf{x}\times\boldsymbol{\Omega})dS + \frac{1}{2}\int_S \mathbf{n}\cdot\boldsymbol{\Omega}(\mathbf{x}\times\mathbf{U})dS.$$

ρ is the air density
V is the control volume (CV)
S is the surface of the CV
$\mathbf{x}$ is the position vector
$\mathbf{n}$ is the outward facing normal from the CV
Ω is the vorticity
$\mathbf{U}$ is the velocity
so the unsteady term directly involves the vorticity and hence the instantanteous forces on the blade

Pitching Airfoils as an Example of Unsteady Flow



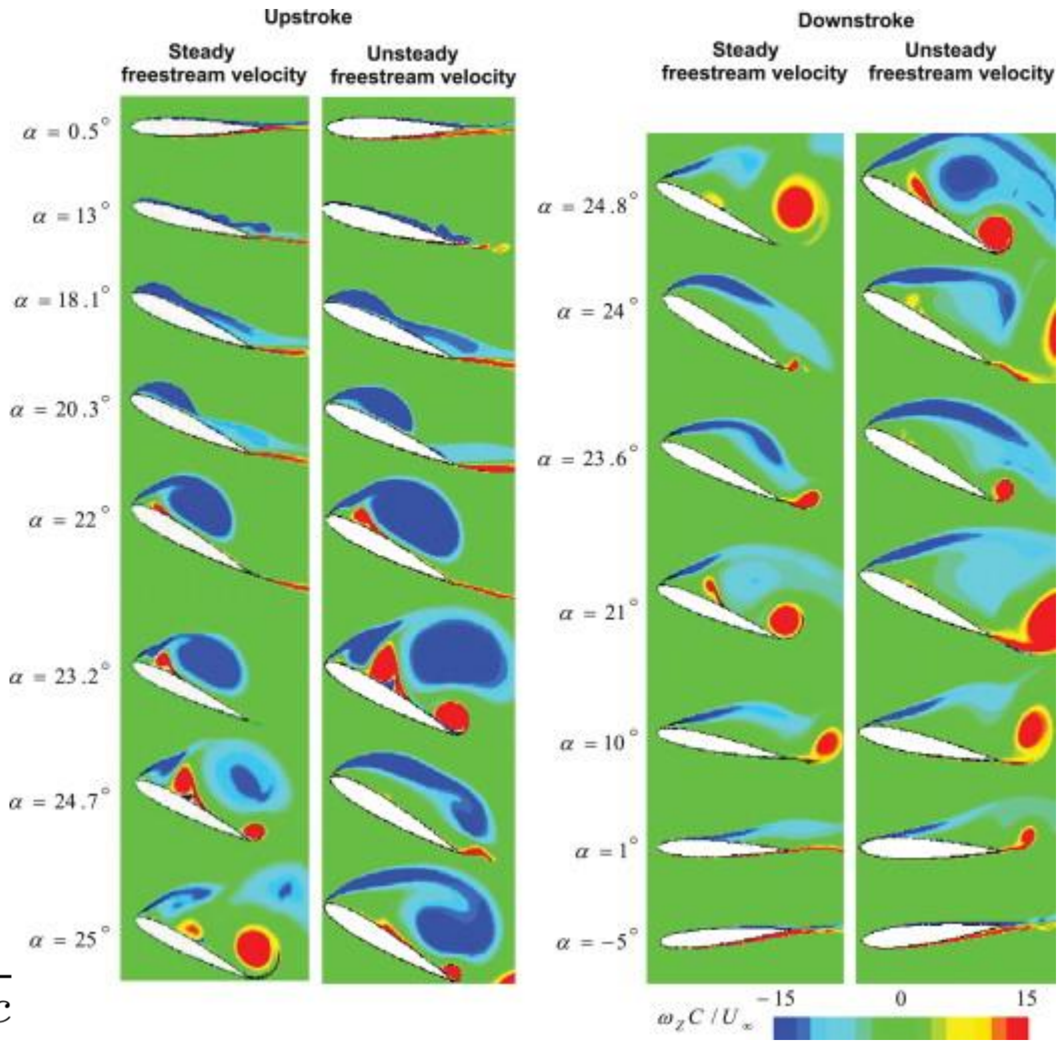
Angle of attack, α, varies with time. The bound and trailing vorticity must also vary with time. Modeling of this effect by Ayman Mohammad (PhD student, UofC) uses a second order equation with delay terms for the bound vorticity

For next slide:

$$\text{Lift coefficient} = \frac{Lift}{\frac{1}{2}\rho V_1^2 c}$$

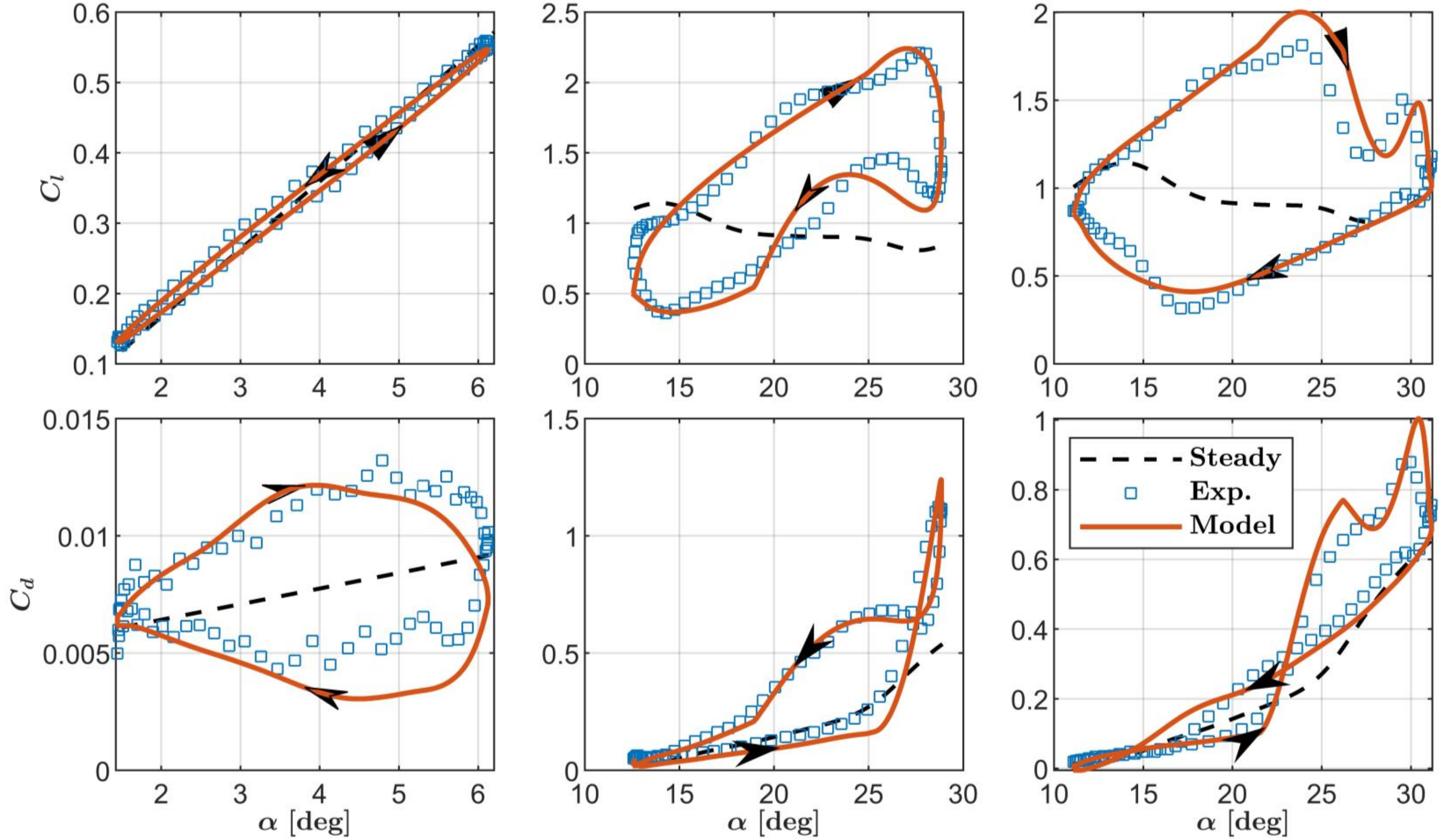
$$\text{Drag coefficient} = \frac{Drag}{\frac{1}{2}\rho V_1^2 c}$$

where c is the airfoil chord



K. Gharali, D.A. Johnson, J. Fluids and Structures, 42, 228-244, 2013.

Forces on Sinusoidally Pitching Airfoil



(a)Case 4 (a)Case 5 (a)Case 6

(a) $\alpha = 4^\circ + 2^\circ \sin(0.1t)$ (b) $\alpha = 20^\circ + 8^\circ \sin(0.15t)$
(c) $\alpha = 20^\circ + 10^\circ \sin(0.076t)$, t is time

Where to Go?

How to combine the dynamic stall model with the unsteady impulse formulations of momentum and angular momentum?

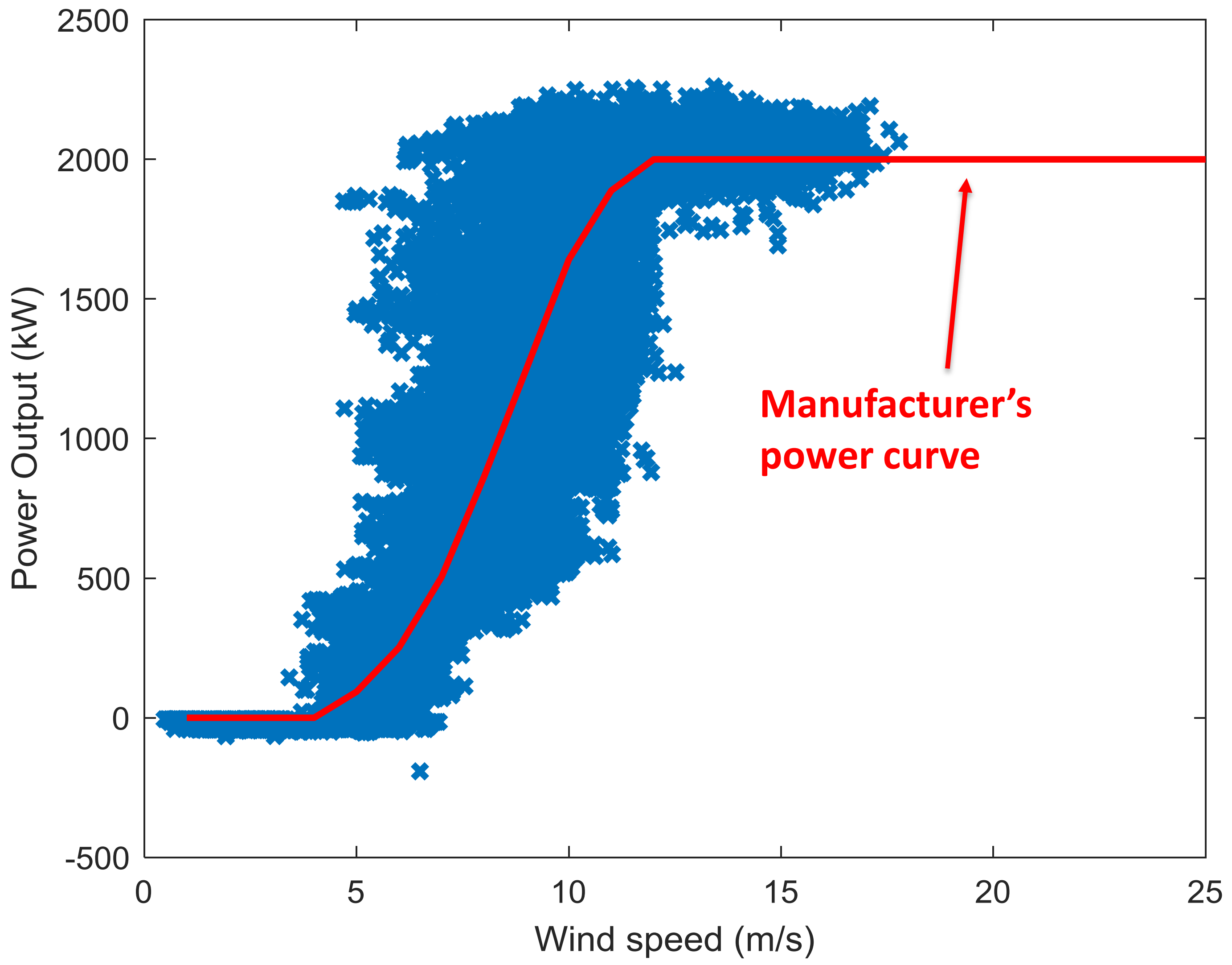
Can this achieve a model that is fast enough to be used in real time control of wind turbines?

Can an improved knowledge of unsteady aerodynamics improve the long-term average power performance of wind turbines?

One day in the life of T5

Output power versus wind speed

Both averaged over 1 second

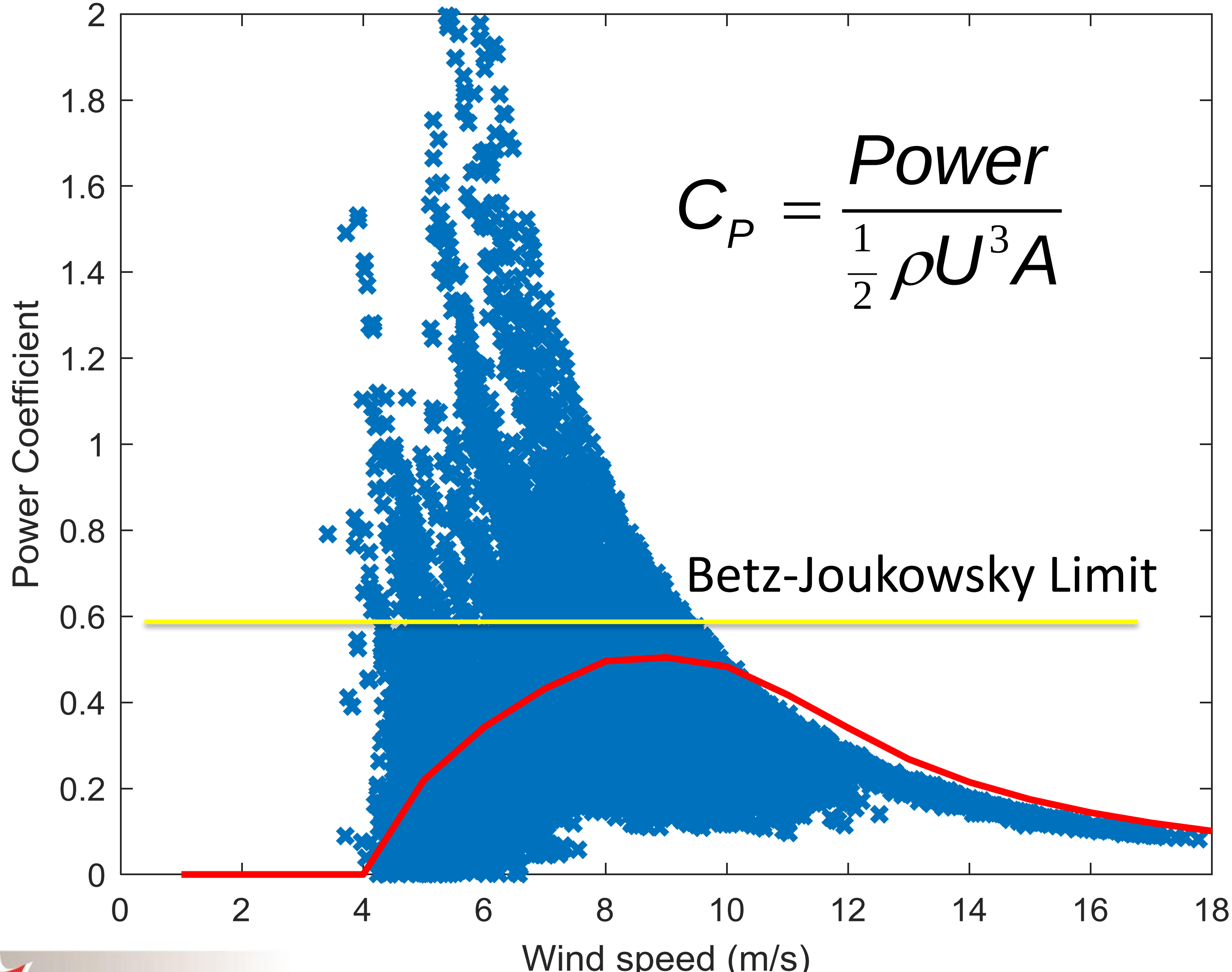


One day in the life of T5

Power coefficient, C_P, versus wind speed

Both averaged over 1 second

ρ – air density
U – wind speed
A – rotor area



$$C_P = \frac{Power}{\frac{1}{2}\rho U^3 A}$$