Summer School on Surgery and the Classification of Manifolds: Lectures and Minilectures

Monday:

9:00–10:00 *Introduction to surgery* (Diarmuid Crowley)

Definition of structure set, the action of $\mathrm{hAut}(X)$ on $\mathcal{S}(X)$, present surgery exact sequence (without defining L-groups or normal invariants), definition of surgery, handlebodies, Morse theory, simply-connected theorem

10:30–11:00 *Handlebody theory* (Allison Miller and Sean Sanford)

Define surgery on an embedding $S^p \times D^{n-p} \hookrightarrow M^n$. Define attaching a $p+1$-handle on an embedding $S^p \times D^{n-p} \hookrightarrow \partial W^{n+1}$. Discuss handles and dual handles. Define a handlebody presentation of a manifold. Give lots of low-dimensional examples.

11:00–11:30 *Morse theory* (Mauricio Bustamante and Alexei Kudryashov)

Define a Morse function on a manifold. State the Morse Lemma. Define the index of a critical point. Discuss elementary cobordisms and the relation to Morse theory. Turn a manifold upside down. Deduce that a Morse function gives a handlebody presentation. See Milnor's paper *A procedure for killing the homotopy groups of differentiable manifolds.*

11:45–12:45 *s/h-cobordism theorem* (Qayum Khan)

Simple homotopy equivalence, Whitehead torsion, state s-cobordism theorem, outline proof of h-cobordism theorem, applications.

2:00–3:00 *Normal maps and Pontrjagin–Thom construction* (Diarmuid Crowley)

Normal map, normal bordism, stable vector bundles, $SO(n)$, SO and BSO, Thom spaces and spectra, the Pontrjagin-Thom isomorphism, computation of framed normal bordism (for a fixed bundle), computation of $\mathcal{N}_*$ and Ω_*^{SO}, degree one.

3:30–4:30 *Topological manifolds* (Jim Davis)

$TOP(n)$, TOP and $BTOP$, statement of topological transversality and Pontryagin-Thom for bordism of topological manifolds, state topological invariance of Whitehead torsion, define topological surgery exact sequence

Tuesday:

9:00–10:00 *Spherical fibrations* (Diarmuid Crowley)

Spherical fibrations and stable spherical fibrations, $G(n)$, G and BG (homotopy groups and what they classify), local coefficients, Poincaré duality and Poincaré complexes, existence and uniqueness of the Spivak normal fibration, identification of normal invariants: $\mathcal{N}(X) \equiv [X, G/O]$

10:30–11:00 *Microbundles and classifying spaces of tangent bundles* (Fedor Manin and Aliaksandra Yarosh)

Define a topological and/or a PL microbundle. Define the microtangent bundle of a topological and of a PL manifold. Define when two microbundles are isomorphic and stably isomorphic. State the theorem that the stable isomorphism classes are in bijective correspondence with homotopy classes of maps to classifying spaces. You should also state Kister's theorem that microbundles are bundles, at least in the topological category.

11:00–11:30 *Poincaré duality and local coefficients* (Maggie Miller and Bena Tshishiku)

Give enough definitions to carefully state Poincaré duality: for a closed oriented connected manifold X^n with fundamental group G, and for a $\mathbb{Z}G$-module M,

$$H^i(X; M) \cong H_{n-i}(X; M)$$

Give examples. Generalize to non-orientable manifolds and/or manifolds with boundary.

11:45–12:45 *Surgery below the middle dimension* (Jim Davis)

The effect of surgery on homotopy and homology, surgery kernels, regular homotopy classes of immersions, the Whitney embedding theorem, surgery below the middle dimension

2:00–3:00 *L-groups* (Qayum Khan)

Rings with involution R, symmetric and quadratic forms, definition of $L_{2q}(R)$, signature, some computations of $L_{2q}(R)$, symmetric and quadratic formations, definition of $L_{2q+1}(R)$, some computations of $L_{2q+1}(R)$

3:30–4:30 *Surgery in the middle dimension* (Qayum Khan)

Equivariant intersection form, equivariant self-intersection, the Whitney trick, definition of the surgery obstruction map $\sigma\colon \mathcal{N}(X) \to L_n(\pi_1(X))$, proof of the fundamental theorem of the surgery for n even.

Wednesday:

9:00–10:00 *The surgery exact sequence (SES)* (Jim Davis)

Wall realization, the action of $L_{n+1}(\pi_1(X))$ on $\mathcal{S}(X)$, proof of exactness of the SES, extension of the SES to the left, the SES for various cases: rel boundary, not rel. boundary, smooth and TOP, the $\pi - \pi$ theorem

10:30–11:00 *Signature and the signature theorem* (Bradley Burdick and Demetre Kazaras)

The goals are to define the signature of a closed, oriented $4k$-dimensional manifold (showing that it is independent of the choice of a basis), show that it is zero on a boundary of a compact manifold, and state the Hirzebruch signature theorem $\text{sign}(M^{4k}) = L_k(p_1, \ldots, p_k)[M]$.

11:00–11:30 *Milnor's paper on exotic spheres* (Ramesh Kasilingam and Jens Reinhold)

The goal is to discuss Milnor's paper *On manifolds homeomorphic to the 7-sphere.*

11:45–12:45 *Exotic spheres* (Diarmuid Crowley)

The J-homomorphism, $\text{Coker}(J)$, the order of bP_{4k}, the Kervaire invariant, $\pi_n(TOP/O) \cong \Theta_n$ is finite, application to smoothing theory.

2:00–3:00 *The homotopy type of G/TOP* (Qayum Khan)

The homotopy groups of G/TOP via the TOP SES for S^n, localisation of spaces, the 2-local homotopy type of G/TOP, the odd-local homotopy type of G/TOP, splitting along submanifolds, $\mathcal{S}^{TOP}(\mathbb{C}P^n)$

3:30–4:30 *The rational homotopy type of manifolds* (Donald Stanley)

Rational Poincaré complexes, rational homotopy type, Sullivan's theorem characterizing the rational homotopy type of simply-connected closed manifolds of dimension greater than four.

Thursday:

9:00–10:00 *Existence of manifold structures* (Diarmuid Crowley)

Non-reducible SNFs, total surgery obstruction, the spherical space form problem

10:30–11:00 *Arf invariant* (Csaba Nagy and Calvin Woo)

The goal is to define the Arf invariant of quadratic form, to show, at least in outline, that the Arf invariant and rank classify nonsingular quadratic forms over $\mathbb{F}_2$. Ilustrate that the 2-torus has a normal framing with nontrivial Arf invariant. One reference is Browder's book *Surgery on simply-connected manifolds*.

11:00–11:30 *Smoothing theory* (Lukas Buggisch and Jan Steinebrunner)

The goal is to state the Fundamental Theorem of Smoothing Theory (see, for example, `http://www.indiana.edu/~jfdavis/notes/smoothing.pdf`) and to indicate the connections with exotic spheres. Note that there are three possible smoothing theories: smoothing topological manifolds, smoothing PL-manifolds, and PL-ing topological manifolds.

11:45–12:45 *The Borel and Novikov Conjectures* (Jim Davis)

Aspherical manifolds, the Borel Conjecture and the Gromov-Lawson Conjecture, the smooth and TOP structure sets of T^n, Higher signatures and the Novikov conjecture, the Novikov conjecture for $\pi_1 = \mathbb{Z}$

2:00–3:00 *Assembly I* (Ian Hambleton)

The L-spectrum, Poincaré duality $[X, G/TOP] \cong H_*(X;\mathbb{L})$, the assembly map $H_*(X;\mathbb{L}) \to L_*(\mathbb{Z}\pi_1 X)$, the factorization of the surgery obstruction map through the assembly map

3:30–4:30 *Assembly II* (Jim Davis)

The Borel and Novikov conjectures via assembly, characteristic class formulas for the surgery obstruction map, the Farrell-Jones conjecture.

Friday:

9:00–10:00 *Higher index theory and the analytic assembly map* (Guoliang Yu)

Higher index theory of elliptic operators and the analytic assembly map. The Baum-Connes conjecture in both the algebraic and analytic settings.

10:30–11:30 *The Dirac operator and positive scalar curvature* (Guoliang Yu)

The Dirac operator and positive scalar curvature. Nonexistence of positive scalar curvature metrics for certain aspherical manifolds. Secondary invariants and the size of moduli space of positive scalar curvature metrics.

11:45–12:45 *The Novikov conjecture and rigidity of manifolds* (Guoliang Yu)

The analytic approach to the Novikov conjecture. Rigidity and nonrigidity of manifolds.