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Reliability-constrained hydropower valuation

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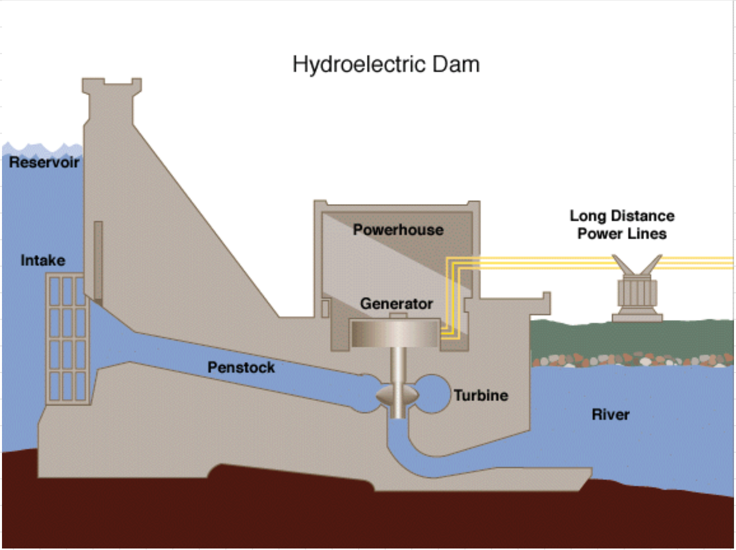
The physical setting

Hydropower facility



Things can sometimes go wrong

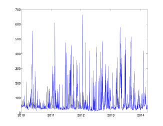
Calgary,
June 2013

Hydropower, dams and rivers

The **marketer's** goal of maximizing revenue must be balanced with the **hydro scheduler's** imperative to operate within the constraints.

Hydropower revenue

- Long-term contracts, 'spot' power markets, ancillary power
- Forward/futures contracts for (imperfect) hedging



Physical factors

- Inflow uncertainty
- Minimum/maximum flow requirements (downstream usage/risk tolerance)
- Turbine performance
- Ice!

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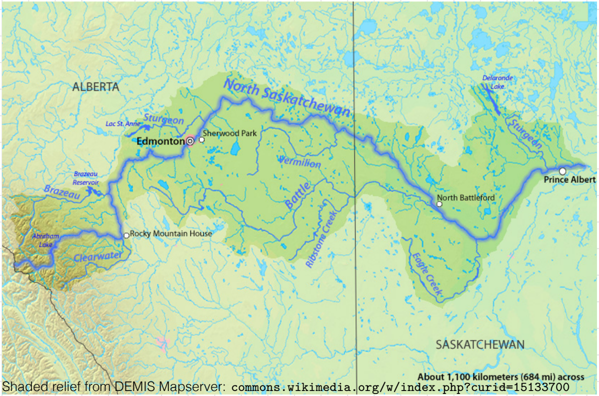
Bighorn

On the North Saskatchewan River

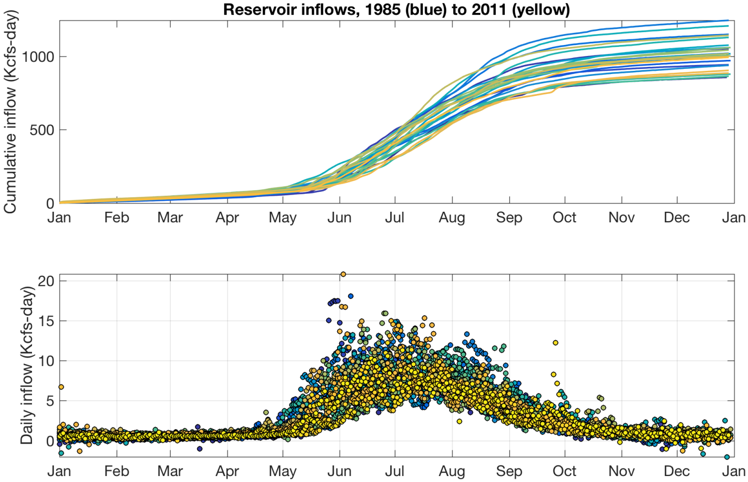


Shaded relief from DEMIS Mapserver: `commons.wikimedia.org/w/index.php?curid=15133700`

- Feeds from Abraham Lake
- Runs through Edmonton, eventually reaches Hudson Bay
- Inflows dominated by spring runoff

Bighorn

Twenty seven years of daily inflows at Abraham Lake



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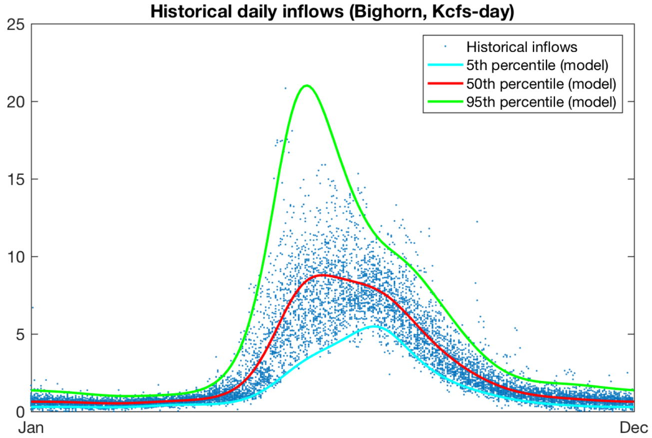
Reliability

Inflow model

- Our initial inflow model is very simple (*i.e. naive*): $\log I_t \sim N\big(\alpha(t), \beta^2(t)\big)$.
- We model the functions α and β using finite Fourier series, and calibrate using MLE.



Outflow restrictions

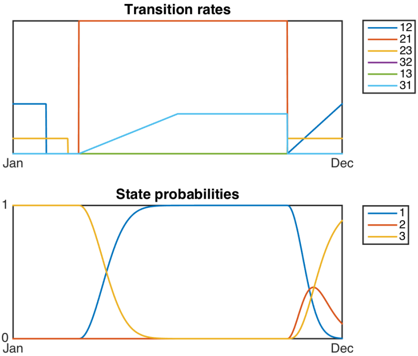
The main sources of constraints in our model relate to outflows.

- Year-round, a minimum flow rate must be maintained in order to supply sufficient water downstream.
- A maximum flow rate needs to be observed so that the river does not burst its banks.
- When ice is forming, flows need to be kept very steady.
- When ice is fully formed, flow may be increased up to a (lower) maximum rate.
- Other restrictions may also apply….

Ice states

The state of ice formation is modelled by a Markov chain, switching between three possible states:

1. 'clear'
2. 'forming'
3. 'solid'.



This means that we enlarge our description of the state to include not only the volume of water in the reservoir, but also the ice state ($i = 1, 2, 3$), and our transition equation involves the Markov switching rates p_t^{ij} from state j to state i, which vary through time (resulting in the unconditional probabilities shown above).

SDP for reliability

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Evolution of the (simple) system state

- In our simplest setting, the system state consists of the ***volume of water in the reservoir*** (V_t), which must be in the interval $[V_{\text{min}}, V_{\text{max}}]$.
- At each time t our control variable u_t is the ***volume of water to flow*** in between t and $t+1$. We will have (possibly time- and state-dependent) constraints on u_t: $u_t \in U_t$.
- If the reservoir receives a random ***inflow volume*** I_t in the interval $[t, t+1]$, then the state evolves according to

$$V_{t+1} = V_t + I_t - u_t.$$

SDP for reliability

We define the reservoir reliability, for a given time horizon T, as:

$$R(t,V) = \sup_{u_t^T \in \mathcal{U}_t^T} \mathbb{P}\Big[V_{t+1},\dots,V_T \in [V_{\min},V_{\max}] \Big| V_t = V\Big]$$
$$= \sup_{u_t^T \in \mathcal{U}_t^T} \mathbb{E}\left[\prod_{k=t+1}^{T} g(V_k) \Big| V_t = V,\right]$$

where $g = \chi_{[V_{\min},V_{\max}]}$, and $\mathcal{U}_t^T$ is the set of admissible feedback controls $u_t^T = \big(u_t(\cdot),\dots,u_{T-1}(\cdot)\big)$.

If we define $R(T,V) := g(V)$, we can determine the functions $R(t,\cdot)$ recursively using the DPP.

SDP for reliability

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DPP

For each t, we define the intermediate function

$$S(t,w) := \mathbb{E}\big[R(t+1, w+I)\big], \quad w \in \mathbb{R}.$$

Then we have, for $t = T-1, T-2, \ldots,$

$$R(t,V) = \sup_{u \in \mathcal{U}_t} S(t, V-u).$$

If the density of I_t is given by the function $f(t,\cdot)$, then we have

$$R(t,V) = \max_{u \in U_t} \int R(t+1, V+x-u) f(t,x) dx.$$

SDP for reliability

Computation

- In order to compute the functions $R_i(t,V)$, we have to ***discretize***, and to this end we create a discrete set of reservoir levels $V_{\text{min}} = V_0, \dots, V_M = V_{\text{max}}$, and assume R_i to be linear between them.
- We also define the intermediate function

$$S_i(t,w) := \int_0^\infty R_i(t, w+x) f(t,x) dx.$$

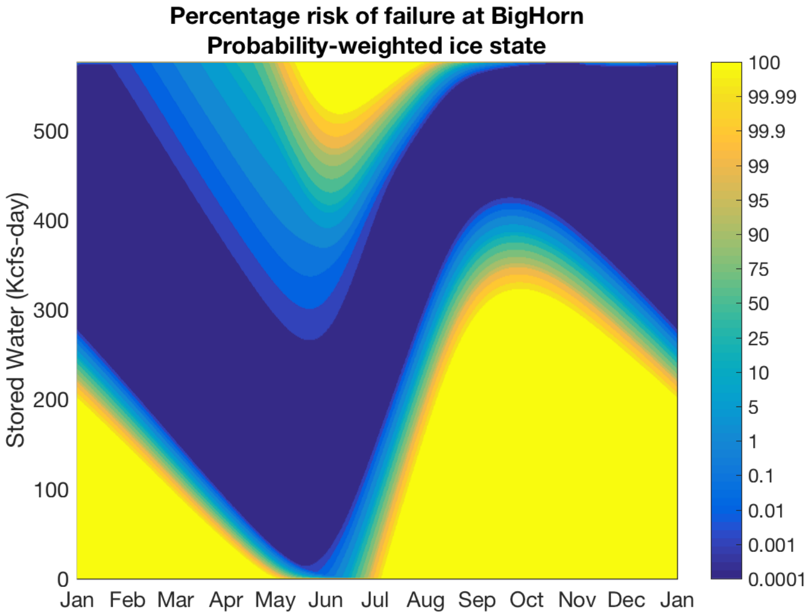
  Note that $R_i(t,V) = \max_{u \in U_t} \sum_j p_t^{ij} S_j(t, V-u)$.
- We create an extended grid for w, and approximate R and S by creating linear interpolants R_i^h and S_i^h on their respective grids.
- Then we have

$$S_i^h(t, W_k) := \int_0^\infty R_i^h(t, W_k + x) f_{t-1}(x) dx,$$

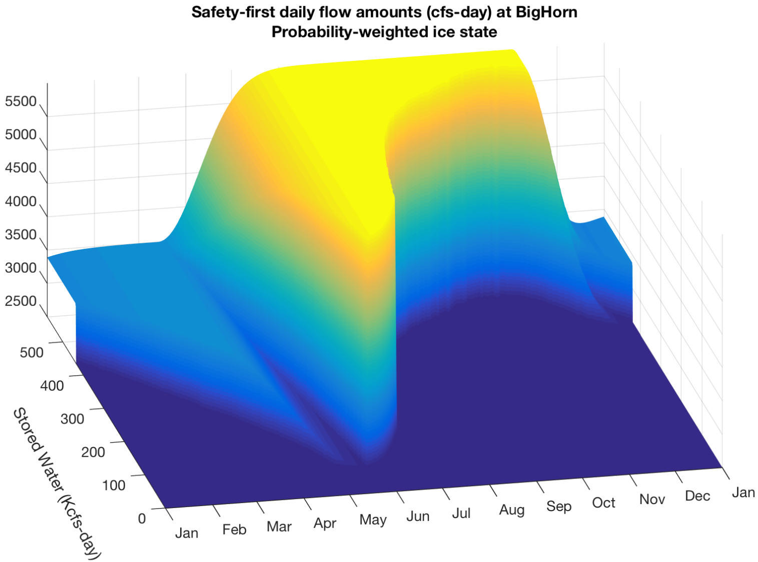
  (computed using discrete convolution) and

$$R_i^h(t, V_k) := \max_{u \in U_t} \sum_j p_t^{ij} S_j^h(t, V_k - u).$$

Results


Percentage risk of failure at BigHorn
Probability-weighted ice state
Stored Water (Kcfs-day)
0
100
200
300
400
500
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan
100
99.99
99.9
99
95
90
75
50
25
10
5
1
0.1
0.01
0.001
0.0001

Results


Safety-first daily flow amounts (cfs-day) at BigHorn
Probability-weighted ice state
5500
5000
4500
4000
3500
3000
2500
500
400
300
200
100
0
Stored Water (Kcfs-day)
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Jan

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Revenue

Revenue

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Here we use a simplified revenue model, in which all power generated is sold on spot markets, with daily average flows determined a day ahead.

Reliability-constrained revenue optimization

- We set an minimum reliability level (i.e. a maximum acceptable probability that constraints will be violated even if a 'safety-first' flow strategy is adopted from that point on).
- We use a standard stochastic optimal control problem to maximize revenue, and to determine the corresponding flow strategy, in regions where the reliability constraint is satisfied.
- The optimal strategy is the one that maximizes revenue until the reliability falls below the minimum level. At this point, the optimal strategy is the safety-first one, until the reliability measure recovers.

Note that this approach differs to some extent from chance-constrained optimization.

Revenue

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- Alberta prices are constrained to be between \$0 and \$1000/MWh.
- We create an empirical CDF F from historical (forecast) prices, and use this to create 'standardized' prices:

$$p_t = \Phi^{-1}\big(F(P_t)\big),$$

  where Φ is the CDF of a standard normal r.v.
- The dynamics of p_t are then modelled using a seasonally-varying AR(1) process:

$$p_{t+1} = \alpha p_t + \beta(t) + \sigma(t) Z_t,$$

  where β and σ are trigonometric polynomials, and $Z_t \sim N(0, 1)$.
- The optimal hydropower value function H is a function of time, p and V (the level of water in the reservoir), i (the ice state) and time.
- Expectations with respect to the dynamics of p are computed using Fourier (cosine) expansions and the FFT.

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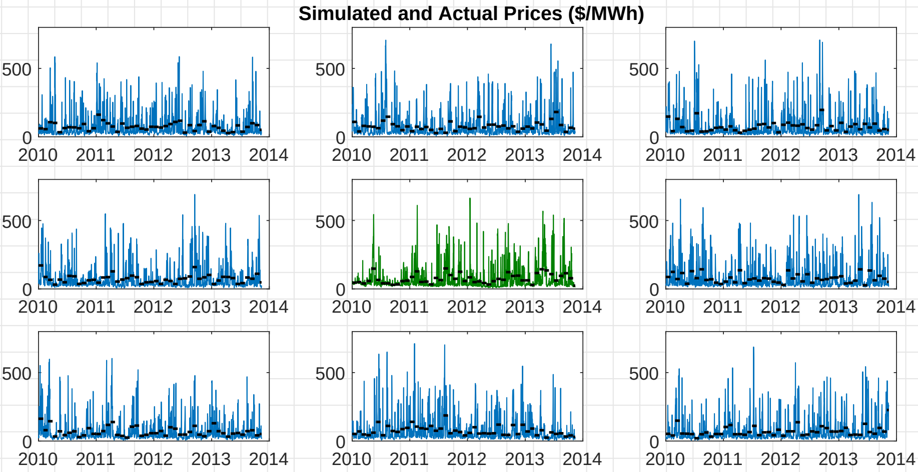
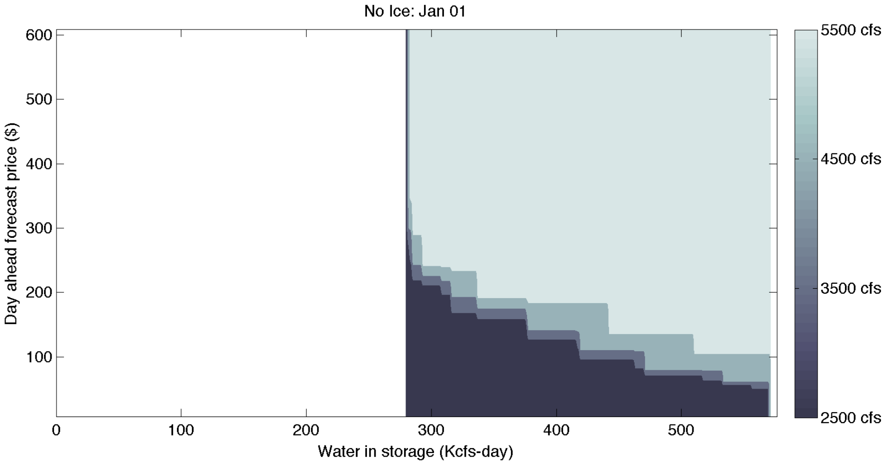
Revenue

Alberta daily average forecast power prices with simulations



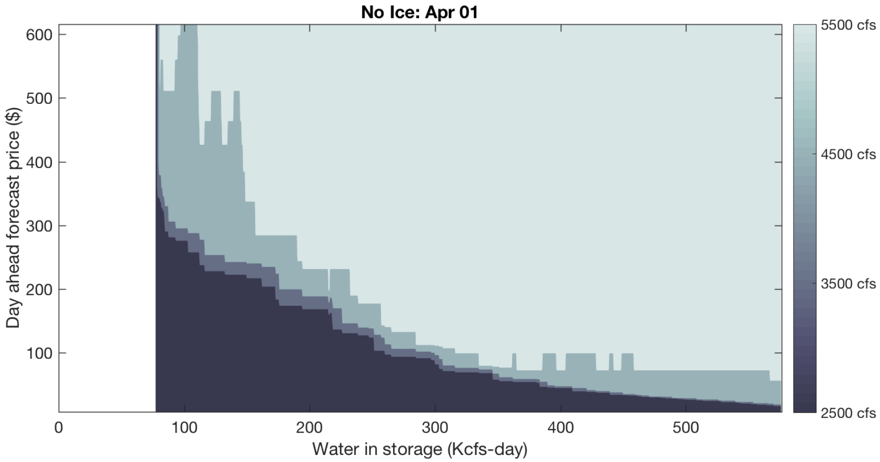
Results

Optimal flow strategies



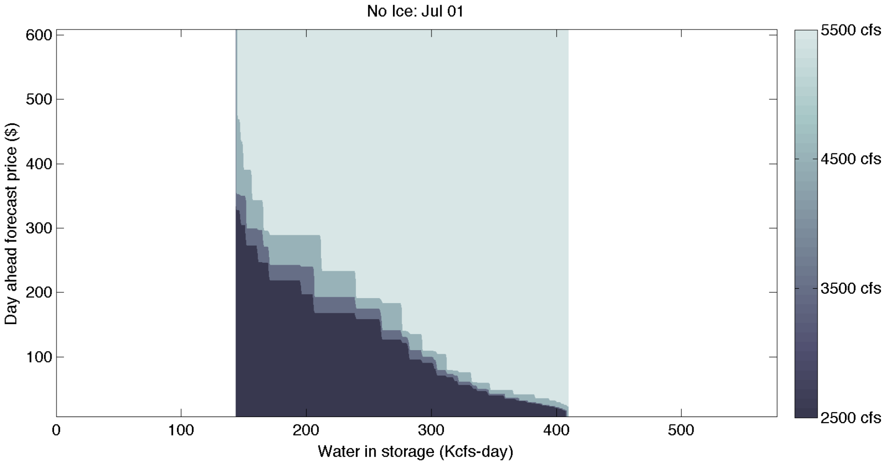
Results

Optimal flow strategies



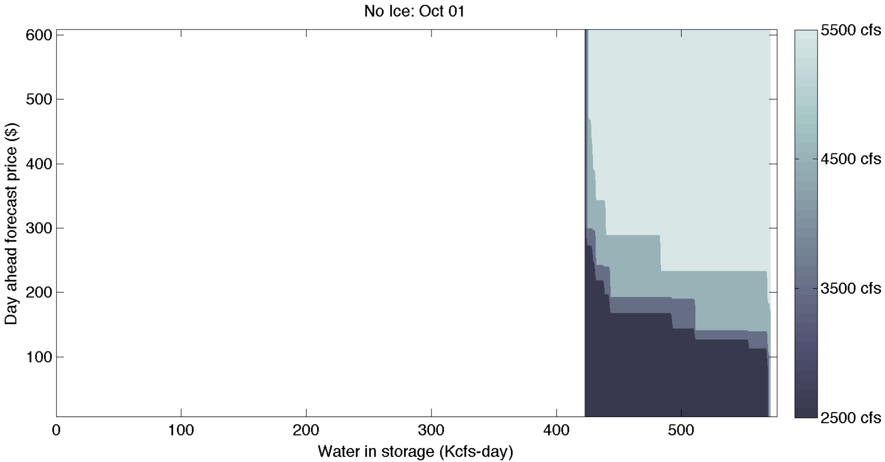
Results

Optimal flow strategies



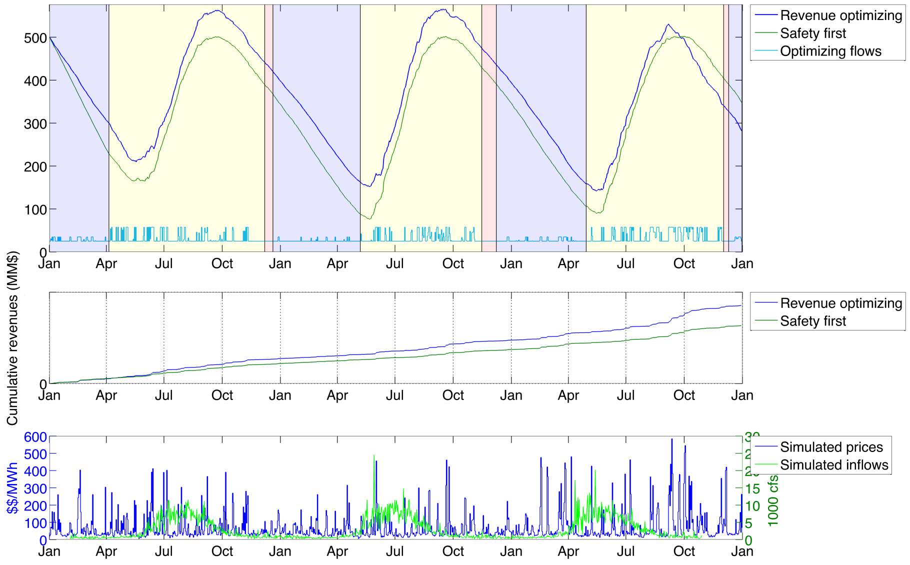
Results

Optimal flow strategies



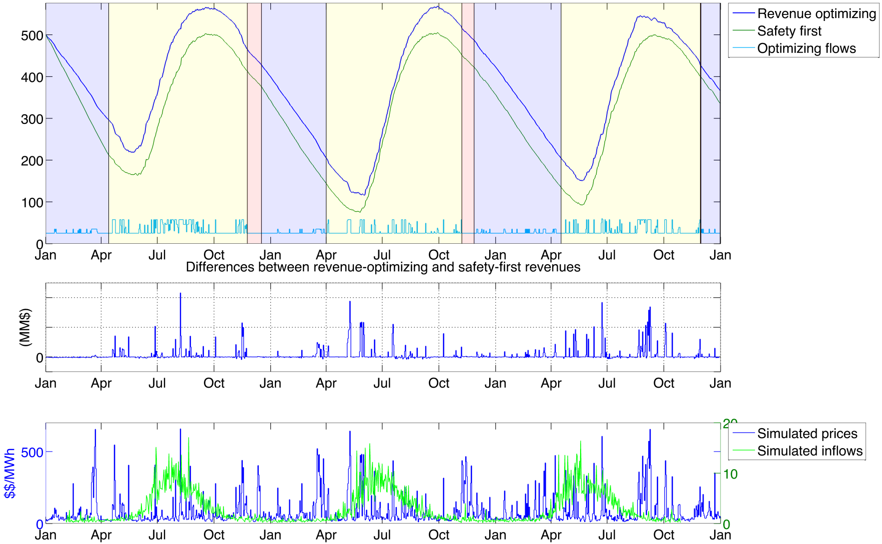
Results

Simulated reservoir operation



Results

Simulated reservoir operation



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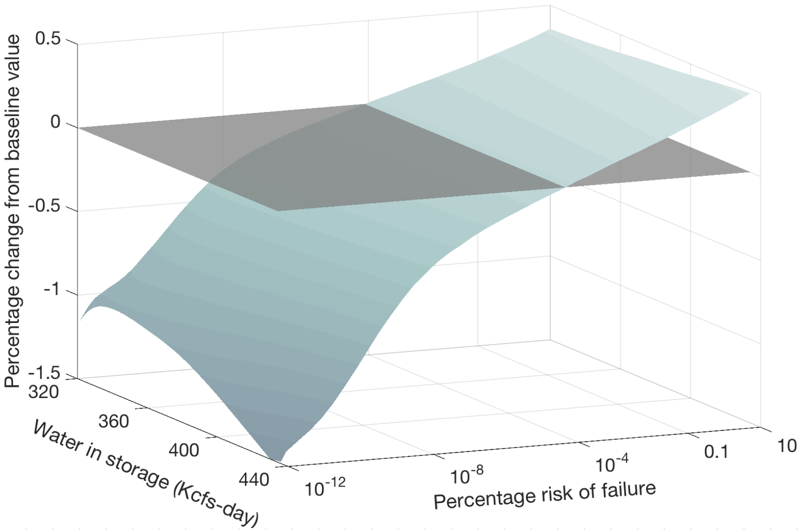
Results

The cost of certainty



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Conclusion

Conclusions

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The model can (needs to) be refined and extended in several directions.

- Improve the inflow model: enlarge the system state to include information on which future inflows can be conditioned (e.g. previous day's flows, hydrological forecasts).
- Improve the inflow model: capture the possibility of extreme inflows.
- Improve the calibration: take account of measurement error.
- Include constraints on the ***rate of change*** in outflows.
- Apply the model to ***systems*** of linked reservoirs.
- Incorporate alternative reliability measures, perhaps in a true chance-constrained framework.
- Incorporate the possibility of hedging using forward contracts of various tenors.

Thank you for your attention!

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References

Bayón, L., Suárez, P., Matías, J., and Taboada, J. (2009). Influence of forecasting electricity prices in the optimization of complex hydrothermal systems. *Journal of Computational and Applied Mathematics*, 232(2):262 – 274.

Carmona, R. and Ludkovski, M. (2010). Valuation of energy storage: an optimal switching approach. *Quantitative Finance*, 10(4):359–374.

Chen, Z. and Forsyth, P. A. (2008). Pricing hydroelectric power plants with/without operational restrictions: a stochastic control approach. *Nonlinear Models in Mathematical Finance. Nova Science*, 1(9).

Näsäkkälä, E. and Keppo, J. (2008). Hydropower with financial information. *Applied Mathematical Finance*, 15(5):1–27.

Pereira, M. V. F. and Pinto, L. M. V. G. (1985). Stochastic optimization of a multireservoir hydroelectric system: A decomposition approach. *Water Resources Research*, 21(6):779–792.

Pereira, M. V. F. and Pinto, L. M. V. G. (1991). Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming*, 52(1):359–375.

Powell, W. B. (2007). *Approximate Dynamic Programming: Solving the curses of dimensionality*. John Wiley & Sons.

Rebennack, S., Pardalos, P. M., Pereira, M. V. F., and Iliadis, N. A., editors (2010a). *Handbook of power systems I*. Springer.

Rebennack, S., Pardalos, P. M., Pereira, M. V. F., and Iliadis, N. A., editors (2010b). *Handbook of Power Systems II*. Springer.

Wallace, S. W. and Fleten, S.-E. (2003). Stochastic programming models in energy. In Ruszczyński, A. and Shapiro, A., editors, *Handbooks in operations research and management science, Volume 10*, chapter 10, pages 637–677. Elsevier.

Yakowitz, S. (1982). Dynamic programming applications in water resources. *Water Resources Research*, 18(4):673–696.

Zhao, G. and Davison, M. (2009a). Optimal control of hydroelectric facility incorporating pump storage. *Renewable Energy*, 34(4):1064 – 1077.

Zhao, G. and Davison, M. (2009b). Valuing hydrological forecasts for a pumped storage assisted hydro facility. *Journal of hydrology*, 373(3):453–462.