Pacific Institute for the Mathematical Sciences

Emergent Research: The PIMS Postdoctoral Fellow Seminar

November 25, 2020
9:30 AM Pacific / 10:30 AM Mountain / 11:30 AM Central
Zoom

PIMS is launching a new lecture series! Every three weeks, you will have the opportunity to connect with emerging research in the mathematical sciences from a PIMS Postdoctoral Fellow. PIMS PDFs are amongst the top young researchers in Canada, and this is an excellent opportunity to learn about them, and their work.

Zafer Selcuk Aygin, PhD

PIMS Postdoctoral Fellow

Relations Between $\triangle + \cdots + \triangle + 3\triangle + \cdots + 3\triangle$ and $\square + \cdots + \square + 3\square + \cdots + 3\square$

Abstract: For non-negative integers *a*, *b* and *n*, let

$$t(a,b;n) = \#\left\{(x_1,\ldots,x_a,y_1,\ldots,y_b) \in \mathbb{Z}^{a+b} \mid n = \tfrac{x_1(x_1-1)}{2} + \cdots + \tfrac{x_a(x_a-1)}{2} + 3\tfrac{y_1(y_1-1)}{2} + \cdots + 3\tfrac{y_b(y_b-1)}{2}\right\}$$

and

$$N(a,b;n) = \#\{(x_1,\ldots,x_a,y_1,\ldots,y_b) \in \mathbb{Z}^{a+b} \mid n = x_1^2 + \cdots + x_a^2 + 3y_1^2 + \cdots + 3y_b^2\}.$$

By works of Bateman and Knopp & Adiga, Cooper and Han, it is known that for (*a*, *b*) = (1, 0), (2, 0), (3, 0), (4, 0), (5, 0), (6, 0), (7, 0), and (3, 1) we have

$$\frac{t(a,b;n)}{N(a,b;8n+a+3b)} = \frac{2}{2^{a+b-2} + 2^{(a+b-2)/2}\cos(\pi(a+3b)/4) + 1}$$

for all $n \in \mathbb{N}$. Moreover, for (a, b) = (8, 0) and (2, 2), Baruah, Cooper and Hirschhorn has proven that

$$\frac{t(a,b;n)}{N(a,b;8n+a+3b) - N(a,b;(8n+a+3b)/4)} = \frac{2}{2^{a+b-2} + 2^{(a+b-2)/2}}$$

for all $n \in \mathbb{N}$.

Variations of such identities were observed by many mathematicians including Baruah, Bateman, Cooper, Dastovski, Hirschhorn, Knopp, Sun and Williams. However, it seems that for bigger values of *a* and *b* these ratios start to fluctuate when *n* varies.

In this work we investigate the limiting cases of similar ratios when *a* + *b* is an even integer greater than *4*, $a \geq 2$ *and* $b \geq 0$. We show that previously known examples are special cases of an asymptotic relation between *N (a, b; n) and t (a, b; n).* We achieve our results by finding certain modular identities which relates generating functions of *t (a, b; n)* to *N (a, b; n).* This is a joint work with Amir Akbary (University of Lethbridge).

Bio: Zafer Selcuk Aygin completed his Ph.D. in mathematics at Carleton University in 2016. He was a research fellow at Nanyang Technological University, Singapore until 2018 and is currently holding a PIMS Post-Doctoral Fellowship at the University of Lethbridge and the University of Calgary on a joint appointment, supported by the Pacific Institute for Mathematical Sciences.

Registration: https://www.pims.math.ca/seminars/PIMSPDF