CENTRE
DE RECHERCHES
MATHÉMATIQUES

October 23, 2024

SMS 2024: Director's report

The subject of the 63rd edition of the *Séminaire de mathématiques supérieures* was *Flows and Variational Methods in Riemannian and Complex Geometry: Classical and Modern Methods*. The summer school was hosted by the **CRM** in Montréal on June 3-14, 2024, and was organized by **Vestislav Apostolov** (UQAM), **Eleonora Di Nezza** (Sorbonne Université), **Pengfei Guan** (McGill University), **Spiro Karigiannis** (University of Waterloo), **Julien Keller** (UQAM), **Alina Stancu** (Concordia University) and **Valentino Tosatti** (New York University).

The school featured ten minicourses on recent advances in Riemannian and Kähler geometry, and related problems in geometric analysis and algebraic geometry. The first week focused on the existence of Kähler metrics and the celebrated Yau-Tian-Donaldson conjecture. Main topics included geometric invariant theory, K-stability, extremal Kähler metrics and Monge-Ampère equations. The second week was concerned with geometric flows in Riemannian and Kähler geometry, such as the mean curvature, the Ricci, and the Kähler-Ricci flows. Various phenomena and applications have been discussed, including the study of singularities and blowup analysis, bounds on the density of minimal cones and uniformization problems in complex geometry.

The school was attended by a diverse and enthusiastic group of more than 75 participants from North America, Europe and Asia. The organizers reported that students have actively participated in discussions, both during and after the lectures, which created a particularly stimulating atmosphere at the school.

Séminaire de mathématiques supérieures would like to thank the organizers of the SMS 2024 for their hard work that made this school possible. We are also grateful to **CRM** and its staff, notably to **Sakina Benhima**, for hosting the event and providing the technical assistance. The SMS acknowledges the continuous financial and logistical support of the **CRM**, **ISM**, **Fields Institute**, **PIMS**, **SLMath**, as well as of the **Université de Montréal.** I would like to thank all these institutions for their contributions. I would also like to express my gratitude to the members of the SMS steering board for their work and dedication.

Attached please find a detailed scientific and organizational report. I thank again the organizers for taking the time to prepare this document.

Best regards,

Iosif Polterovich,
Director, Séminaire de mathématiques supérieures

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SMS 2024 - Flows and Variational Methods in Riemannian and Complex Geometry: Classical and Modern Methods

June 3 - 14, 2024

Organizers

- Vestislav Apostolov (Université du Québec à Montréal)
- Eleonora Di Nezza (Sorbonne Université)
- Pengfei Guan (McGill University)
- Spiro Karigiannis (University of Waterloo)
- Julien Keller (Université du Québec à Montréal)
- Alina Stancu (Concordia University)
- Valentino Tosatti (New York University)

Overview

The last few years have seen spectacular progress on manifolds of various difficult geometric problems. Probably the most familiar of these problems to the general mathematical audience is the solution of the Poincaré conjecture, but the general idea of taking a canonical variational flow to obtain geometric objects of interest of course has its origins much earlier, notably in the work of Yau, of Donaldson, and of several others in the 1970s and 1980s. The more recent Ricci flow solution of the Poincaré conjecture has stimulated a whole series of major breakthroughs in related areas of geometry, notably in complex and Kähler geometry, for example in the solution of the Kähler-Einstein existence problem for Fano varieties which is linked to stability notions in algebraic geometry. Beyond the deep geometrical questions, partial differential equations and flows appear as an essential tool to solve conjectures and have their own interests.

From a general perspective, the SMS school intended to present recent developments in both Riemannian and Kähler geometry around the notion of curvature seen as a tool to describe and understand the geometry of the underlying objects.

The timing of the school could not have been better. At the end of April, Simon Brendle received the 2024 Breakthrough prize for transformative contributions to differential geometry including results on the Ricci flow and the mean curvature flow. Consequently, these topics attracted a lot of attention. The audience was extra keen to follow the mini courses delivered by Bamler, Choi and Haslhofer which touched on key notions of Brendle's work.

The **first week of the SMS** was centered around the question of existence of Kähler metrics with special curvature properties and the famous Yau-Tian-Donaldson conjecture. These important metrics can also be seen as critical points of certain energy functionals that are natural from a symplectic point of view, using moment maps. Recently, variational methods applied to these functionals and the use of pluripotentials and valuations have provided a link between the existence of such metrics and K-stability in terms of non-Archimedean geometry. Moreover, it has been discovered that many of the potential spaces in the Kähler setting have a rich metric geometry, turning the variational problems into problems of infinite dimensional convex optimization, yielding existence results for solutions of various complex Monge–Ampère type equations**.** The first week aims focused at introducing these results and techniques by providing lectures on the following items:

- GIT stability, moment maps, K-stability and its variants
- Finite energy pluripotential theory, geometry of the space of Kähler potentials
- Variational methods and Kähler-Einstein metrics
- Non-Archimedean Kähler geometry and K-stability

Brief description of the mini-courses for week 1

The metric completion of the space of Kahler metrics and applications (T. Darvas, Maryland)

In the 1950's Calabi recognized that the geometric structure of the space of Kahler metrics plays an important role in the study of canonical metrics. Later Mabuchi, Semmes and Donaldson found the precise infinite dimensional Riemannian metric that unlocks the key properties of this space. In this minicourse it was given a detailed study of the Mabuchi metric completion of the space of Kahler metrics and shown how it naturally connects with the finite energy pluripotential theory initiated by Guedj and Zeriahi. It was also discussed the variational approach to Kahler-Einstein metrics, in relation to the Yau-Tian-Donalson conjectures.

Geometric invariant theory, moment maps, K-stability and its variants (R. Dervan, Glasgow)

Geometric invariant theory (GIT) was introduced by Mumford as a construction of quotients of algebraic varieties by group actions. It has since found many applications to both the construction of moduli spaces and to their study. The course introduced GIT with an emphasis on its links with symplectic geometry, which arise through the theory of moment maps and symplectic reduction. Furthermore, the lectures explained how GIT has motivated the Yau-Tian-Donaldson conjecture, which relates the existence of canonical Kähler metrics to the notion of K-stability.

K-stability and non-Archimedean geometry (M. Jonsson, Michigan)

K-stability is a condition of algebro-geometric nature which appears in the Yau-Tian-Donaldson conjecture as a criterion for the existence of particular Kähler metrics on projective complex manifolds. The course explained how one can think of K-stability in terms of non-Archimedean geometry in the sense of Berkovich.

Weighted extremal Kähler metrics and applications (E. Legendre, Lyon 1)
In recent years, many powerful tools have been developed to investigate the existence problem of Calabi extremal Kähler metrics, particularly regarding the more famous cases of Einstein and constant scalar curvature Kähler metrics. As expected, these tools, including various notions of (algebraic) stability and weighted extremal Kähler metrics, are now used to explore the extremal problem within families of Kähler structures and, in the most understood case, the moduli spaces of stable varieties have been described. This course gave a review the notions of weighted extremal Kähler metrics from the differential point of view of Lahdilli as well as variational aspects following Inoue, Han and Li. Applications of this theory towards the Yau-Tian-Donaldson correspondence have been discussed as well.
Monge-Ampère equations on compact hermitian manifolds (C. Lu, Angers)
After recalling classical results for complex Monge-Ampère equations on Kähler manifolds, it was explained the advances of the last decade on compact hermitian manifolds (existence, uniqueness, degenerations).

The **second week of the SMS** focused on geometric flows, mostly in Riemannian geometry. The evolution equations describing the deformation of geometric objects by curvature are highly nonlinear parabolic differential equations with many technical aspects and they were the object of five minicourses: three of the courses addressed problems in the area of extrinsic flows while two other courses focused on Ricci and, respectively Ricci-Kähler, flow.

Brief Description of the mini-courses for week 2

Mean curvature flow through singularities (R. Haslhofer, Toronto)
Mean curvature flow is the most natural evolution equation in extrinsic geometry and, historically, the first geometric curvature flow that was studied. This course introduced the mean curvature flow of surfaces with a focus on the analysis of singularities, hence presenting many general concepts and methods, such as monotonicity formulas, epsilon-regularity, weak solutions, and blowup analysis that are of great importance in the analysis of a wide range of geometric PDEs.
Ancient mean curvature flow (K. Choi, KIAS)
Ancient mean curvature flows are solutions to the mean curvature flow that exist for all negative time. Since curvature flows are parabolic differential equations, parabolic Liouville theorems allow the classification of ancient flows. The latter is extremely important as ancient flows capture a lot of information on the singularities of the flow. The course presented the latest developments in this area with a lot of emphasis on the intuition behind the results and potential for generalizations.
A mean curvature flow approach to density of minimal cones (Lu Wang, Yale)
In this mini-course, the focus was on the applications of the mean curvature flow to minimal surfaces. This is natural to consider as minimal cones are models for singularities in the area of minimal submanifolds while they are also stationary solutions to the mean curvature flow.

Students were exposed to questions on lower bounds for density of minimal cones and how to utilize mean curvature flow to yield near optimal estimates on the density of minimal cones.

Ricci flows in dimension 4 and higher (R. Bamler, Berkeley)

The course described recent progress on the study of Ricci flow higher dimensions, including a general compactness result for non-collapsed Ricci flows and a structure theory for the limit spaces.

Overview of some recent work of Kähler-Ricci Flow (M-C. Lee, CUHK)

This mini-course gave an overview of recent results on the Kähler-Ricci flow, both in the compact and complete noncompact settings, with applications to uniformization problems in complex geometry, and to the study of Gromov-Hausdorff limit spaces of Kähler manifolds with curvature lower bounds.

Students were actively participating in discussions during and after the lectures, which led to a very stimulating atmosphere. On an anecdotal level, we had to drag the speakers away for lunch because the students were keeping them busy at the blackboards.

ORGANIZERS	AFFILIATION
Apostolov Vestislav	UQAM
Di Nezza Eleonora	Sorbonne
Guan Pengfei	McGill
Karigiannis Spiro	University of Waterloo
Kellller Julien	UQAM
Stancu Alina	Concordia University
Tosatti Valentino	Courant Institute
SPEAKERS	
Bamler Richard	University of California, Berkeley
Choi Kyeongsu	KIAS
Darvas Tamas	University of Maryland
Dervan Ruadhai	University of Glasgow
Haslhofer Robert	University of Toronto
Lee Man Chun	The Chinese University of Hong Kong
Legendre Eveline	Université Claude Bernard Lyon 1
Lu Hoang-Chinh	Université d'Angers
Jonsson Mattias	University of Michigan
Wang Lu	Yale
SMS supported students	
Aylaian Ezra	Duke University
Araujo Roberto	University of Münster
Babu Ivin	The University of Texas at Dallas
Braga Vitor	Indiana University
Cerqueira Gonçaves Guilherme	Toulouse III - Université Paul Sabatier
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Fan Colin	Northwestern University
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Frey Gabriel	ETH Zürich
Fung Cheuk Yan	University of Maryland, College Park
Hou Yu-Chi	University of Maryland, College Park
Huang Jiuzhou	Korea Institute For Advanced Study (KIAS)
Hughes, John	University of Oxford
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Lee Ching Hsien	Math Department of Rutgers University
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Wang Yipeng	Columbia University
Xu Jie	Northeastern University
NSF Supported participants	
Firester Benjy	MIT
Xu Kai	Duke University
Yang Yang	Johns Hopkins University
SMS no financial supp	
Bao Yueheng	University of Science and Technology of China
Hashimoto Yoshinori	Osaka Metropolitan University
Hong Fang	McGill University
Hosseini Meraj	Concordia University
Hyu Jia-Lin	National Taiwan University
Istrati Nicolina	Université d'Angers
Li Yung-Chen	National Taiwan University
Reznikov Jacob	McGIll University
Xiao Xia	UQAM
SLMath supported Students	
Dai Xinle (Clair)	Harvard University
Durham Cole	University of Connecticut
Fan Colin	Northwestern University
Fluck Harry	Cornell University
Friedman Benjamin	University of British Columbia
George Mathew	Ohio State University
Greilhuber Josef	Stanford University
Guelen Muhammed E.	Berlin Mathematical School
Hsiao Ming	National Center for Theoretical Sciences
Jatar Aneesh	University of Hong Kong
Koirala Robert	UC San Diego
Law Michael	Massachusetts Institute of Technology
Li Xinze	University of Toronto
Nguyen Duc	Lehigh University
Romshoo Faisal	University of Waterloo
Shao Guanhua	Rutgers University, New Brunswick
Soundararajan Sidharth	Boston University
Tsiamis Raphael	Columbia University

Valiyakath Nazia	Syracuse University
Virgin Bryce	University of South Florida
Wang Ying	University of Michigan
Womack Michael	UC Irvine

WEEK 1 SEMAINE 1	Monday June 3 / Lundi 3 Juin	Tuesday June 4 / Mardi 4 Juin	Wednesday June 5 / Mercredi 5 Juin	Thursday June 6 / Jeudi 6 Juin	Friday June 7 / Vendredi 7 Juin
09:00	Opening / Ouverture	*E. Legendre* **Weighted extremal Kähler metrics and applications (2)**	*M. Jonsson* **K-stability and non-Archimedean geometry (3)**	*C. Lu* **Monge-Ampère equations on compact hermitian manifolds (4)**	*R. Dervan* **Geometric invariant theory, moment maps, K-stability and its variants (5)**
09:15	*T. Darvas* **The metric completion of the space of Kahler metrics and applications (1)**				
09:30					
09:45					
10:00		Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café
10:15	Coffee break / Pause café				
10:30		*T. Darvas* **The metric completion of the space of Kahler metrics and applications (2)**	*E. Legendre* **Weighted extremal Kähler metrics and applications (3)**	*M. Jonsson* **K-stability and non-Archimedean geometry (4)**	*C. Lu* **Monge-Ampère equations on compact hermitian manifolds (5)**
10:45	*R. Dervan* **Geometric invariant theory, moment maps, K-stability and its variants (1)**				
11:00					
11:15					
11:30					
11:45	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas
12:00					
12:15					
12:30					
12:45					
13:00					
13:15					
13:30	*C. Lu* **Monge-Ampère equations on compact hermitian manifolds (1)**	*R. Dervan* **Geometric invariant theory, moment maps, K-stability and its variants (2)**	*T. Darvas* **The metric completion of the space of Kahler metrics and applications (3)**	*E. Legendre* **Weighted extremal Kähler metrics and applications (4)**	*M. Jonsson* **K-stability and non-Archimedean geometry (5)**
13:45					
14:00					
14:15					
14:30	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café
14:45					
15:00	*M. Jonsson* **K-stability and non-Archimedean geometry (1)**	*C. Lu* **Monge-Ampère equations on compact hermitian manifolds (2)**	*R. Dervan* **Geometric invariant theory, moment maps, K-stability and its variants (3)**	*T. Darvas* **The metric completion of the space of Kahler metrics and applications (4)**	*E. Legendre* **Weighted extremal Kähler metrics and applications (5)**
15:15					
15:30					
15:45					
16:00	Break / Pause	Break / Pause	Break / Pause	Break / Pause	Break / Pause
16:15	*E. Legendre* **Weighted extremal Kähler metrics and applications (1)**	*M. Jonsson* **K-stability and non-Archimedean geometry (2)**	*C. Lu* **Monge-Ampère equations on compact hermitian manifolds (3)**	*R. Dervan* **Geometric invariant theory, moment maps, K-stability and its variants (4)**	*T. Darvas* **The metric completion of the space of Kahler metrics and applications (5)**
16:30					
16:45					
17:00					
17:15					
17:30				Wine & Cheese / Vin & Fromage	
17:45					
18:00					
18:15					
18:30					

WEEK 2 SEMAINE 2	Monday June 10 / Lundi 10 Juin	Tuesday June 11 / Mardi 11 Juin	Wednesday June 12 / Mercredi 12 Juin	Thursday June 13 / Jeudi 13 Juin	Friday June 14 / Vendredi 14 Juin
09:00	Opening / Ouverture	*R. Haslhofer* **Mean curvature flow through singularities (2)**	*R. Haslhofer* **Mean curvature flow through singularities (3)**	*R. Haslhofer* **Mean curvature flow through singularities (4)**	*R. Haslhofer* **Mean curvature flow through singularities (5)**
09:15	*R. Haslhofer* **Mean curvature flow through singularities (1)**				
09:30					
09:45					
10:00		Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café
10:15	Coffee break / Pause café				
10:30		*L. Wang* **A mean curvature flow approach to density of minimal cones (2)**	*L. Wang* **A mean curvature flow approach to density of minimal cones (3)**	*L. Wang* **A mean curvature flow approach to density of minimal cones (4)**	*R. Bamler* **Ricci flows in dimension 4 and higher (5)**
10:45	*L. Wang* **A mean curvature flow approach to density of minimal cones (1)**				
11:00					
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11:45	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas	Lunch break / Pause repas
12:00					
12:15					
12:30					
12:45					
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13:15					
13:30	*K. Choi* **Ancient mean curvature flow (1)**	*K. Choi* **Ancient mean curvature flow (2)**	*K. Choi* **Ancient mean curvature flow (3)**	*K. Choi* **Ancient mean curvature flow (4)**	*L. Wang* **A mean curvature flow approach to density of minimal cones (5)**
13:45					
14:00					
14:15					
14:30	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café	Coffee break / Pause café
14:45					
15:00	*R. Bamler* **Ricci flows in dimension 4 and higher (1)**	*R. Bamler* **Ricci flows in dimension 4 and higher (2)**	*R. Bamler* **Ricci flows in dimension 4 and higher (3)**	*R. Bamler* **Ricci flows in dimension 4 and higher (4)**	*K. Choi* **Ancient mean curvature flow (5)**
15:15					
15:30					
15:45					
16:00	Break / Pause	Break / Pause	Break / Pause	Break / Pause	Break / Pause
16:15	*M.-C. Lee* **Overview of some recent work of Kähler-Ricci Flow (1)**	*M.-C. Lee* **Overview of some recent work of Kähler-Ricci Flow (2)**	*M.-C. Lee* **Overview of some recent work of Kähler-Ricci Flow (3)**	*M.-C. Lee* **Overview of some recent work of Kähler-Ricci Flow (4)**	*M.-C. Lee* **Overview of some recent work of Kähler-Ricci Flow (5)**
16:30					
16:45					
17:00					
17:15					
17:30				Wine & Cheese / Vin & Fromage	
17:45					
18:00					
18:15					
18:30					

CENTRE DE RECHERCHES MATHÉMATIQUES

2024-EESMS_Suivi financier 1.xlsx

Étiquettes de lignes	Somme de Sum Val. péc.
Frais de séjour	**109 516,04 $**
Graphisme	**534,69 $**
Logistique	**204,39 $**
Revenus d'inscription	**(3 174,60) $**
Traiteur	**8 062,80 $**
Total général	**115 143,32 $**

Répartition ($ Can)				
CIRGET	2 000,00 $			
CRM	17 275,29 $			
ISM	8 637,65 $			
Fields	8 637,65 $			
PIMS	8 637,65 $		**$ US**	
SLMath (20 étudiants)	67 195,30 $	$	48 372,91	
NSF (2 étudiants)	2 759,79 $	$	1 857,01	
Total	**115 143,32 $**		**50 229,92 $**	